

Effective Field Theory and χ pt

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February 1, 2008

Abstract

A brief introduction to the subject of chiral perturbation theory (χ pt) is given, including a discussion of effective field theory and application to the upcoming Bates virtual Compton scattering measurement.

1 Introduction

We have gathered to celebrate the fact that Bates has been delivering beam successfully for twenty five years and to review some of the things which have been learned and which are still to be studied. One thing that *has* changed theoretically during this period is that we now have a new paradigm for analysis of low energy processes such as studied at Bates. I was a student in the 1960's and at that time our goal was to attempt to find a renormalizable field theory which describes all particle interactions with the same sort of success as quantum electrodynamics (QED). In 1967 we went part of the way with development of the Weinberg-Salam theory, which incorporated the weak interaction as a sibling to the electromagnetic. Because the interaction was weak it could be treated via the same perturbative techniques as could its electromagnetic kin and what has resulted is an extremely successful description of all weak and electromagnetic processes.

For the strong interactions a renormalizable picture has also been developed—quantum chromodynamics or QCD. The theory is, of course, deceptively simple on the surface. Indeed the form of the Lagrangian¹

$$\mathcal{L}_{\text{QCD}} = \bar{q}(iD - m)q - \frac{1}{2}\text{tr } G_{\mu\nu}G^{\mu\nu}. \quad (3)$$

is elegant, and the theory is renormalizable. So why are we not satisfied? While at the very largest energies, asymptotic freedom allows the use of perturbative techniques, for those who are interested in making contact with low energy experimental findings there exist at least three fundamental difficulties:

- i) QCD is written in terms of the "wrong" degrees of freedom—quarks

¹Here the covariant derivative is

$$iD_\mu = i\partial_\mu - gA_\mu^a \frac{\lambda^a}{2}, \quad (1)$$

where λ^a (with $a = 1, \dots, 8$) are the SU(3) Gell-Mann matrices, operating in color space, and the color-field tensor is defined by

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g[A_\mu, A_\nu], \quad (2)$$

and gluons—while low energy experiments are performed with hadronic bound states;

- ii) the theory is non-linear due to gluon self interactions;
- iii) the theory is one of strong coupling— $g^2/4\pi \sim 1$ —so that perturbative methods are not practical.

Nevertheless, there has been a great deal of recent progress in making contact between theory and experiment using the technique of "effective field theory", which exploits the chiral symmetry of the QCD interaction. In order to understand how this is accomplished, we shall first review this idea of effective field theory in the simple context of quantum mechanics. Then we show how these ideas can be married via chiral perturbation theory and indicate applications at Bates.

2 Effective Field Theory

The power of effective field theory is associated with the feature that there exist many situations in physics involving *two scales*, one heavy and one light. Then, provided one is working at energies small compared to the heavy scale, it is possible to fully describe the interactions in terms of an "effective" picture, which is written only in terms of the light degrees of freedom, but which fully includes the influence of the heavy mass scale through virtual effects. A number of very nice review articles on effective field theory can be found in ref. [1].

Before proceeding to QCD, however, it is useful to study this idea in the simpler context of ordinary quantum mechanics, in order to get familiar with the concept. Specifically, we examine the question of why the sky is blue, whose answer can be found in an analysis of the scattering of photons from the sun by atoms in the atmosphere—Compton scattering[2]. First we examine the problem using traditional quantum mechanics and consider elastic (Rayleigh) scattering from, for simplicity, single-electron (hydrogen) atoms. The appropriate Hamiltonian is then

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + e\phi \quad (4)$$

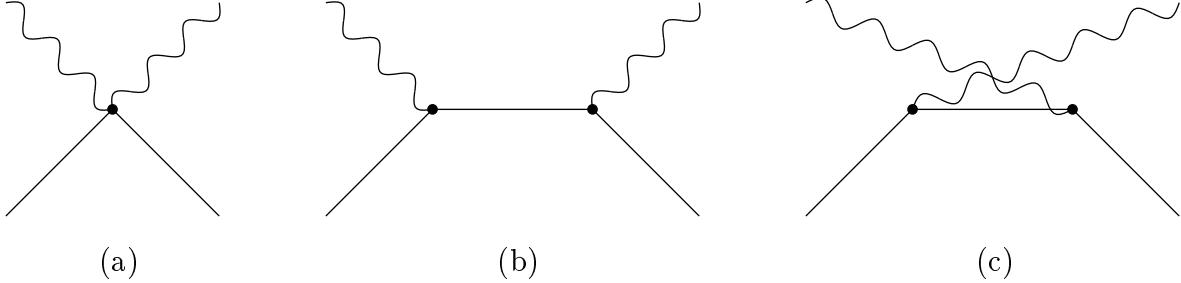


Figure 1: Feynman diagrams for nonrelativistic photon-atom scattering.

and the leading— $\mathcal{O}(e^2)$ —amplitude for Compton scattering is found from calculating the diagrams shown in Figure 1, yielding the familiar Kramers-Heisenberg form

$$\begin{aligned} \text{Amp} = & -\frac{e^2/m}{\sqrt{2\omega_i 2\omega_f}} \left[\hat{\epsilon}_i \cdot \hat{\epsilon}_f^* + \frac{1}{m} \sum_n \left(\frac{\hat{\epsilon}_f^* \cdot < 0 | \vec{p} e^{-i\vec{q}_f \cdot \vec{r}} | n > \hat{\epsilon}_i \cdot < n | \vec{p} e^{i\vec{q}_i \cdot \vec{r}} | 0 >}{\omega_i + E_0 - E_n} \right. \right. \\ & \left. \left. + \frac{\hat{\epsilon}_i \cdot < 0 | \vec{p} e^{i\vec{q}_i \cdot \vec{r}} | n > \hat{\epsilon}_f^* \cdot < n | \vec{p} e^{-i\vec{q}_f \cdot \vec{r}} | 0 >}{E_0 - \omega_f - E_n} \right) \right] \end{aligned} \quad (5)$$

where $|0\rangle$ represents the hydrogen ground state having binding energy E_0 .

Here the leading component is the familiar ω -independent Thomson amplitude and would appear naively to lead to an energy-independent cross-section. However, this is *not* the case. Indeed, by expanding in ω and using a few quantum mechanical identities one can show that, provided that the energy of the photon is much smaller than a typical excitation energy—as is the case for optical photons, the cross section can be written as

$$\frac{d\sigma}{d\Omega} = \lambda^2 \omega^4 |\hat{\epsilon}_f^* \cdot \hat{\epsilon}_i|^2 \left(1 + \mathcal{O} \left(\frac{\omega^2}{(\Delta E)^2} \right) \right) \quad (6)$$

where

$$\lambda = \alpha_{em} \sum \frac{2|z_{n0}|^2}{E_n - E_0} \quad (7)$$

is the atomic electric polarizability, $\alpha_{em} = e^2/4\pi$ is the fine structure constant, and $\Delta E \sim m\alpha_{em}^2$ is a typical hydrogen excitation energy. We note that $\alpha_{em}\lambda \sim a_0^2 \times \frac{\alpha_{em}}{\Delta E} \sim a_0^3$ is of order the atomic volume, as will be exploited below, and that the cross section itself has the characteristic ω^4 dependence

which leads to the blueness of the sky—blue light scatters much more strongly than red[3].

Now while the above derivation is certainly correct, it requires somewhat detailed and lengthy quantum mechanical manipulations which obscure the relatively simple physics involved. One can avoid these problems by the use of effective field theory methods. The key point is that of scale. Since the incident photons have wavelengths $\lambda \sim 5000\text{A}$ much larger than the $\sim 1\text{A}$ atomic size, then at leading order the photon is insensitive to the presence of the atom, since the latter is electrically neutral. If χ represents the wavefunction of the atom then the effective leading order Hamiltonian is simply

$$H_{eff}^{(0)} = \chi^* \left(\frac{\vec{p}^2}{2m} + e\phi \right) \chi \quad (8)$$

and there is *no* interaction with the field. In higher orders, there *can* exist such atom-field interactions and this is where the effective Hamiltonian comes in to play. In order to construct the effective interaction, we demand certain general principles—this Hamiltonian must satisfy fundamental symmetry requirements. In particular H_{eff} must be gauge invariant, must be a scalar under rotations, and must be even under both parity and time reversal transformations. Also, since we are dealing with Compton scattering, H_{eff} should be quadratic in the vector potential. Actually, from the requirement of gauge invariance, it is clear that the effective interaction can utilize \vec{A} only via the electric and magnetic fields, rather than the vector potential itself—

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A}, \quad \vec{B} = \vec{\nabla} \times \vec{A} \quad (9)$$

since these are invariant under a gauge transformation

$$\phi \rightarrow \phi + \frac{\partial}{\partial t}\Lambda, \quad \vec{A} \rightarrow \vec{A} - \vec{\nabla}\Lambda \quad (10)$$

while the vector and/or scalar potentials are not. The lowest order interaction then can involve only the rotational invariants \vec{E}^2 , \vec{B}^2 and $\vec{E} \cdot \vec{B}$. However, under spatial inversion— $\vec{r} \rightarrow -\vec{r}$ —electric and magnetic fields behave oppositely— $\vec{E} \rightarrow -\vec{E}$ while $\vec{B} \rightarrow \vec{B}$ —so that parity invariance rules out any dependence on $\vec{E} \cdot \vec{B}$. Likewise under time reversal invariance

$\vec{E} \rightarrow \vec{E}$, $\vec{B} \rightarrow -\vec{B}$ so such a term is also T-odd. The simplest such effective Hamiltonian must then have the form

$$H_{eff}^{(1)} = \chi^* \chi \left[-\frac{1}{2} c_E \vec{E}^2 - \frac{1}{2} c_B \vec{B}^2 \right] \quad (11)$$

(Terms involving time or spatial derivatives are much smaller.) We know from electrodynamics that $\frac{1}{2}(\vec{E}^2 + \vec{B}^2)$ represents the field energy per unit volume, so by dimensional arguments, in order to represent an energy in Eq. 11, c_E, c_B must have dimensions of volume. Also, since the photon has such a long wavelength, there is no penetration of the atom, so only classical scattering is allowed. The relevant scale must then be atomic size so that we can write

$$c_E = k_E a_0^3, \quad c_B = k_B a_0^3 \quad (12)$$

where we anticipate $k_E, k_B \sim \mathcal{O}(1)$. Finally, since for photons with polarization $\hat{\epsilon}$ and four-momentum q_μ we identify $\vec{A}(x) = \hat{\epsilon} \exp(-iq \cdot x)$, then from Eq. 9, $|\vec{E}| \sim \omega$, $|\vec{B}| \sim |\vec{k}| = \omega$ and

$$\frac{d\sigma}{d\Omega} \propto | \langle f | H_{eff} | i \rangle |^2 \sim \omega^4 a_0^6 \quad (13)$$

as found in the previous section via detailed calculation. This is a nice example of the power of simple effective field theory arguments.

3 Application to QCD: Chiral Perturbation Theory

Now let's apply these ideas to the case of QCD. In this case the invariance we wish to exploit is "chiral symmetry." The idea of "chirality" is defined by the operators

$$\Gamma_{L,R} = \frac{1}{2}(1 \pm \gamma_5) = \frac{1}{2} \begin{pmatrix} 1 & \mp 1 \\ \mp 1 & 1 \end{pmatrix} \quad (14)$$

which project "left-" and "right-handed" components of the Dirac wavefunction via

$$\psi_L = \Gamma_L \psi \quad \psi_R = \Gamma_R \psi \quad \text{with} \quad \psi = \psi_L + \psi_R \quad (15)$$

In terms of these chirality states the quark component of the QCD Lagrangian can be written as

$$\bar{q}(i \not{D} - m)q = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L m q_R - \bar{q}_R m q_L \quad (16)$$

The reason that these chirality states are called left- and right-handed is that in the limit $m \rightarrow 0$ they coincide with quark *helicity* projection operators. With this background, we note that QCD, in the mathematical limit as $m \rightarrow 0$ has the structure

$$\mathcal{L}_{\text{QCD}} \xrightarrow{m=0} \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R \quad (17)$$

and is invariant under *independent* global left- and right-handed rotations

$$q_L \rightarrow \exp(i \sum_j \lambda_j \alpha_j) q_L, \quad q_R \rightarrow \exp(i \sum_j \lambda_j \beta_j) q_R \quad (18)$$

This invariance is called $SU(3)_L \otimes SU(3)_R$ or chiral $SU(3) \times SU(3)$. Continuing to neglect the light quark masses, we see that in a chiral symmetric world one would expect to have sixteen—eight left-handed and eight right-handed—conserved Noether currents

$$\bar{q}_L \gamma_\mu \frac{1}{2} \lambda_i q_L, \quad \bar{q}_R \gamma_\mu \frac{1}{2} \lambda_i q_R \quad (19)$$

Equivalently, by taking the sum and difference we would have eight conserved vector and eight conserved axial vector currents

$$V_\mu^i = \bar{q} \gamma_\mu \frac{1}{2} \lambda_i q, \quad A_\mu^i = \bar{q} \gamma_\mu \gamma_5 \frac{1}{2} \lambda_i q \quad (20)$$

In the vector case, this is just a simple generalization of isospin ($SU(2)$) invariance to the case of $SU(3)$. There exist *eight* ($3^2 - 1$) time-independent charges

$$F_i = \int d^3x V_0^i(\vec{x}, t) \quad (21)$$

and there exist various supermultiplets of particles having identical spin-parity and (approximately) the same mass in the configurations—singlet, octet, decuplet, *etc.* demanded by $SU(3)$ -invariance.

If chiral symmetry were realized in the conventional fashion one would expect there also to exist corresponding nearly degenerate same spin but

opposite parity states generated by the action of the time-independent axial charges $F_i^5 = \int d^3x A_0^i(\vec{x}, t)$ on these states. However, it is known that the axial symmetry is broken spontaneously, whereby Goldstone's theorem requires the existence of eight massless pseudoscalar bosons, which couple derivatively to the rest of the universe[4]. Of course, in the real world such massless 0^- states do not exist, because in the real world exact chiral invariance is broken by the small quark mass terms which we have neglected up to this point. Thus what we have are eight very light (but not massless) pseudo-Goldstone bosons which make up the pseudoscalar octet. Since such states are lighter than their other hadronic counterparts, we have a situation wherein effective field theory can be applied—provided one is working at energy-momenta small compared to the ~ 1 GeV scale which is typical of hadrons, one can describe the interactions of the pseudoscalar mesons using an effective Lagrangian. Actually this has been known since the 1960's, where a good deal of work was done with a *lowest order* effective chiral Lagrangian[5]

$$\mathcal{L}_2 = \frac{F_\pi^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{m_\pi^2}{4} F_\pi^2 \text{Tr}(U + U^\dagger). \quad (22)$$

where the subscript 2 indicates that we are working at two-derivative order or one power of chiral symmetry breaking—*i.e.* m_π^2 . Here $U \equiv \exp(\sum \lambda_i \phi_i / F_\pi)$, where $F_\pi = 92.4$ is the pion decay constant. This Lagrangian is *unique*—if we expand to lowest order in $\vec{\phi}$

$$\begin{aligned} \text{Tr} \partial_\mu U \partial^\mu U^\dagger &= \text{Tr} \frac{i}{F_\pi} \vec{\tau} \cdot \partial_\mu \vec{\phi} \times \frac{-i}{F_\pi} \vec{\tau} \cdot \partial^\mu \vec{\phi} = \frac{2}{F_\pi^2} \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} \\ \text{Tr}(U + U^\dagger) &= \text{Tr}(2 - \frac{1}{F_\pi^2} \vec{\tau} \cdot \vec{\phi} \vec{\tau} \cdot \vec{\phi}) = \text{const.} - \frac{2}{F_\pi^2} \vec{\phi} \cdot \vec{\phi} \end{aligned} \quad (23)$$

we reproduce the free pion Lagrangian, as required,

At the SU(3) level, including an appropriately generalized chiral symmetry breaking term, there is even predictive power—one has

$$\frac{F_\pi^2}{4} \text{Tr} \partial_\mu U \partial^\mu U^\dagger = \frac{1}{2} \sum_{j=1}^8 \partial_\mu \phi_j \partial^\mu \phi_j + \dots \quad (24)$$

$$\frac{F_\pi^2}{4} \text{Tr} 2B_0 m(U + U^\dagger) = \text{const.} - \frac{1}{2} (m_u + m_d) B_0 \sum_{j=1}^3 \phi_j^2$$

$$- \frac{1}{4}(m_u + m_d + 2m_s)B_0 \sum_{j=4}^7 \phi_j^2 - \frac{1}{6}(m_u + m_d + 4m_s)B_0 \phi_8^2 + \dots \quad (25)$$

where B_0 is a constant and m is the quark mass matrix. We can then identify the meson masses as

$$\begin{aligned} m_\pi^2 &= 2\hat{m}B_0 \\ m_K^2 &= (\hat{m} + m_s)B_0 \\ m_\eta^2 &= \frac{2}{3}(\hat{m} + 2m_s)B_0, \end{aligned} \quad (26)$$

where $\hat{m} = \frac{1}{2}(m_u + m_d)$ is the mean light quark mass. This system of three equations is *overdetermined*, and we find by simple algebra

$$3m_\eta^2 + m_\pi^2 - 4m_K^2 = 0. \quad (27)$$

which is the Gell-Mann-Okubo mass relation and is well-satisfied experimentally[6]. Expanding to fourth order in the fields we also reproduce the well-known and experimentally successful Weinberg $\pi\pi$ scattering lengths[7]

$$a_0^0 = \frac{7m_\pi^2}{32\pi F_\pi^2}, \quad a_0^2 = -\frac{m_\pi^2}{16\pi F_\pi^2}, \quad a_1^1 = \frac{m_\pi^2}{24\pi F_\pi^2} \quad (28)$$

However, when one attempts to go beyond tree level in order to unitarize the results, divergences arise and that is where the field stopped at the end of the 1960's. The solution, as pointed out ten years later by Weinberg[8] and carried out by Gasser and Leutwyler[9], is to absorb these divergences in phenomenological constants, just as done in QED. A new wrinkle in this case is that the theory is nonrenormalizable in that the forms of the divergences are *different* from the terms that one started with. That means that the form of the counterterms that are used to absorb these divergences must also be different, and Gasser and Leutwyler wrote down the most general counterterm Lagrangian that one can have at one loop, which involves *four-derivative* interactions

$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i \mathcal{O}_i = L_1 \left[\text{tr}(D_\mu U D^\mu U^\dagger) \right]^2 + L_2 \text{tr}(D_\mu U D_\nu U^\dagger) \cdot \text{tr}(D^\mu U D^\nu U^\dagger)$$

$$\begin{aligned}
& + L_3 \text{tr}(D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger) + L_4 \text{tr}(D_\mu U D^\mu U^\dagger) \text{tr}(\chi U^\dagger + U \chi^\dagger) \\
& + L_5 \text{tr}(D_\mu U D^\mu U^\dagger (\chi U^\dagger + U \chi^\dagger)) + L_6 \left[\text{tr}(\chi U^\dagger + U \chi^\dagger) \right]^2 \\
& + L_7 \left[\text{tr}(\chi^\dagger U - U \chi^\dagger) \right]^2 + L_8 \text{tr}(\chi U^\dagger \chi U^\dagger + U \chi^\dagger U \chi^\dagger) \\
& + i L_9 \text{tr}(F_{\mu\nu}^L D^\mu U D^\nu U^\dagger + F_{\mu\nu}^R D^\mu U^\dagger D^\nu U) + L_{10} \text{tr}(F_{\mu\nu}^L U F^{R\mu\nu} U^\dagger)
\end{aligned} \tag{29}$$

where the covariant derivative is defined via

$$D_\mu U = \partial_\mu U + \{A_\mu, U\} + [V_\mu, U] \tag{30}$$

the constants $L_i, i = 1, 2, \dots, 10$ are arbitrary (not determined from chiral symmetry alone) and $F_{\mu\nu}^L, F_{\mu\nu}^R$ are external field strength tensors defined via

$$F_{\mu\nu}^{L,R} = \partial_\mu F_\nu^{L,R} - \partial_\nu F_\mu^{L,R} - i[F_\mu^{L,R}, F_\nu^{L,R}], \quad F_\mu^{L,R} = V_\mu \pm A_\mu. \tag{31}$$

Now just as in the case of QED the bare parameters L_i which appear in this Lagrangian are not physical quantities. Instead the experimentally relevant (renormalized) values of these parameters are obtained by appending to these bare values the divergent one-loop contributions—

$$L_i^r = L_i - \frac{\gamma_i}{32\pi^2} \left[\frac{-2}{\epsilon} - \ln(4\pi) + \gamma - 1 \right] \tag{32}$$

By comparing predictions with experiment, Gasser and Leutwyler were able to determine empirical values for each of these ten parameters. Typical results are shown in Table 1, together with the way in which they were determined. The important question to ask at this point is why stop at order four derivatives? Clearly if two-loop amplitudes from \mathcal{L}_2 or one-loop corrections from \mathcal{L}_4 are calculated, divergences will arise which are of six-derivative character. Why not include these? The answer is that the chiral procedure represents an expansion in energy-momentum. Corrections to the lowest order (tree level) predictions from one-loop corrections from \mathcal{L}_2 or tree level contributions from \mathcal{L}_4 are $\mathcal{O}(E^2/\Lambda_\chi^2)$ where $\Lambda_\chi \sim 4\pi F_\pi \sim 1$ GeV is the chiral scale[10]. Thus chiral perturbation theory is a *low energy* procedure. It is only to the extent that the energy is small compared to the chiral scale that it makes sense to truncate the expansion at the one-loop (four-derivative)

Coefficient	Value	Origin
L_1^r	0.65 ± 0.28	$\pi\pi$ scattering
L_2^r	1.89 ± 0.26	and
L_3^r	-3.06 ± 0.92	$K_{\ell 4}$ decay
L_5^r	2.3 ± 0.2	F_K/F_π
L_9^r	7.1 ± 0.3	π charge radius
L_{10}^r	-5.6 ± 0.3	$\pi \rightarrow e\nu\gamma$

Table 1: Gasser-Leutwyler counterterms and the means by which they are determined.

level. Realistically this means that we deal with processes involving $E < 500$ MeV, and for such reactions the procedure is found to work very well.

In fact Gasser and Leutwyler, besides giving the form of the $\mathcal{O}(p^4)$ chiral Lagrangian, have also performed the one loop integration and have written the result in a simple algebraic form. Users merely need to look up the result in their paper and, despite having ten phenomenological constants the theory is quite predictive. An example is shown in Table 2, where predictions are given involving quantities which arise using just two of the constants— L_9, L_{10} . The table also reveals an interesting dilemma—one solid chiral prediction, that for the charged pion polarizability, is possibly violated, although this is far from clear since there are three experimental results here, only one of which is in disagreement. This represents a serious challenge to the chiral predictions (and therefore to QCD!) and should be the focus of future experimental work. However, there are no Bates implications and, because of space limitations, we shall have to be content to stop here. Interested readers, however, can find applications to this and other systems in a number of review articles[16].

4 χ pt and Bates

For application at Bates it is important to note that the same ideas can be applied within the sector of meson-nucleon interactions, although with a bit more difficulty. Again much work has been done in this regard[17], but there

Reaction	Quantity	Theory	Experiment
$\pi^+ \rightarrow e^+ \nu_e \gamma$	$h_V(m_\pi^{-1})$	0.027	0.029 ± 0.017 [11]
$\pi^+ \rightarrow e^+ \nu_e e^+ e^-$	r_V/h_V	2.6	2.3 ± 0.6 [11]
$\gamma \pi^+ \rightarrow \gamma \pi^+$	$(\alpha_E + \beta_M) (10^{-4} \text{ fm}^3)$	0	1.4 ± 3.1 [12]
	$\alpha_E (10^{-4} \text{ fm}^3)$	2.8	6.8 ± 1.4 [13]
			12 ± 20 [14]
			2.1 ± 1.1 [15]

Table 2: Chiral Predictions and data in radiative pion processes.

remain important challenges[18]. Writing the lowest order chiral Lagrangian at the SU(2) level is straightforward—

$$\mathcal{L}_{\pi N} = \bar{N}(i \not{D} - m_N + \frac{g_A}{2} \not{\mu} \gamma_5)N \quad (33)$$

where g_A is the usual nucleon axial coupling in the chiral limit, the covariant derivative $D_\mu = \partial_\mu + \Gamma_\mu$ is given by

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}u^\dagger(V_\mu + A_\mu)u - \frac{i}{2}u(V_\mu - A_\mu)u^\dagger, \quad (34)$$

and u_μ represents the axial structure

$$u_\mu = iu^\dagger \nabla_\mu U u^\dagger \quad (35)$$

Expanding to lowest order we find

$$\begin{aligned} \mathcal{L}_{\pi N} = & \bar{N}(i \not{\partial} - m_N)N + g_A \bar{N} \gamma^\mu \gamma_5 \frac{1}{2} \vec{\tau} N \cdot \left(\frac{i}{F_\pi} \partial_\mu \vec{\pi} + 2 \vec{A}_\mu \right) \\ & - \frac{1}{4F_\pi^2} \bar{N} \gamma^\mu \vec{\tau} N \cdot \vec{\pi} \times \partial_\mu \vec{\pi} + \dots \end{aligned} \quad (36)$$

which yields the Goldberger-Treiman relation, connecting strong and weak couplings of the nucleon system[19]

$$F_\pi g_{\pi NN} = m_N g_A \quad (37)$$

Using the present best values for these quantities, we find

$$92.4\text{MeV} \times 13.05 = 1206\text{MeV} \quad \text{vs.} \quad 1189\text{MeV} = 939\text{MeV} \times 1.266 \quad (38)$$

and the agreement to better than two percent strongly confirms the validity of chiral symmetry in the nucleon sector. Actually the Goldberger-Treiman relation is only strictly true at the unphysical point $g_{\pi NN}(q^2 = 0)$ and one *expects* about a 1% discrepancy to exist. An interesting "wrinkle" in this regard is the use of the so-called Dashen-Weinstein relation, which takes into account lowest order SU(3) symmetry breaking, to predict this discrepancy in terms of corresponding numbers in the strangeness changing sector[20].

Another successful application at tree level involves threshold charged pion photoproduction and the Kroll-Ruderman term[21], which arises from the feature that, since the pion must be derivatively coupled, there exists a $\bar{N}N\pi^\pm\gamma$ contact interaction which dominates threshold charged pion photoproduction. Here what is measured is the s-wave or E_{0+} multipole, defined via

$$\text{Amp} = 4\pi(1 + \mu)E_{0+}\vec{\sigma} \cdot \hat{\epsilon} + \dots \quad (39)$$

where $\mu = m_\pi/M$. The chiral symmetry prediction is[22]

$$\begin{aligned} E_{0+} &= \pm \frac{1}{4\pi(1 + \mu)} \frac{eg_A}{\sqrt{2}F_\pi} \left(1 \mp \frac{\mu}{2}\right) = \frac{eg_A}{4\sqrt{2}F_\pi} \begin{pmatrix} 1 - \frac{3}{2}\mu & \pi^+ \\ -1 + \frac{1}{2}\mu & \pi^- \end{pmatrix} \\ &= \begin{cases} +26.3 \times 10^{-3}/m_\pi & \pi^+ n \\ -31.3 \times 10^{-3}/m_\pi & \pi^- p \end{cases}, \end{aligned} \quad (40)$$

which is in excellent agreement with the present experimental results, as shown in Table 3.

However, any realistic approach must also involve loop calculations as well as the use of a Foldy-Wouthuysen transformation in order to assure proper power counting. This approach goes under the name of heavy baryon chiral perturbation theory (HB χ pt) and interested readers can find a compendium of such results in the review article[28]. For our purposes we shall have to be content to examine just two applications. One is neutral pion photoproduction. In this case the Kroll-Ruderman term is absent and the chiral expansion of the E_{0+} threshold amplitude begins at order μ and a heavy baryon HB χ pt calculation by Bernard, Kaiser, and Meissner found an important loop contribution which had been omitted in the previous PCAC-based approach[29].

Quantity	Expt.
$E_{0+}(\gamma p \rightarrow \pi^+ n)$	$(+27.9 \pm 0.5) \times 10^{-3}/m_\pi$ [23]
	$(+28.8 \pm 0.7) \times 10^{-3}/m_\pi$ [24]
	$(+27.6 \pm 0.3) \times 10^{-3}/m_\pi$ [25]
$E_{0+}(\gamma n \rightarrow \pi^- p)$	$(-31.4 \pm 1.3) \times 10^{-3}/m_\pi$ [23]
	$(-32.2 \pm 1.2) \times 10^{-3}/m_\pi$ [26]
	$(-31.5 \pm 0.8) \times 10^{-3}/m_\pi$ [27]

Table 3: Experimental values for E_{0+} multipoles in charged pion photoproduction.

The correct chiral prediction at $\mathcal{O}(\mu^2)$ was found to be[30]

$$E_{0+} = \frac{eg_A}{8\pi M} \mu \left\{ 1 - \left[\frac{1}{2}(3 + \kappa_p) + \left(\frac{M}{4F_\pi} \right)^2 \right] \mu + \mathcal{O}(\mu^2) \right\} \quad (41)$$

where the term in M^2 signifies the “new” chiral loop contribution. However, comparison with experiment is tricky because of the existence of isotopic spin breaking in the pion and nucleon masses, so that there are *two* thresholds—one for $\pi^0 p$ and the second for $\pi^+ n$ —only 7 MeV apart. When the physical masses of the pions are used recent data from both Mainz and from Saskatoon agree with the chiral prediction. However, there are concerns about the convergence of the chiral expansion, which reads $E_{0+} = C(1 - 1.26 + 0.59 + \dots)$. There also exist chiral predictions for threshold p-wave amplitudes which are in good agreement with experiment, as shown in Table 4, and for which the convergence is expected to be rapid.

Finally exists a chiral symmetry prediction for the reaction $\gamma n \rightarrow \pi^0 n$

$$E_{0+} = -\frac{eg_A}{8\pi M} \mu^2 \left\{ \frac{1}{2} \kappa_n + \left(\frac{M}{4F_\pi} \right)^2 \right\} + \dots = 2.13 \times 10^{-3}/m_\pi \quad (42)$$

However, the experimental measurement of such an amplitude involves considerable challenge, and must be accomplished either by use of a deuterium target with the difficult subtraction of the proton contribution and of meson exchange contributions or by use of a ${}^3\text{He}$ target. Neither of these are straightforward although some limited data already exist[33].

Our final example involves an experiment at Bates—measurement of the *generalized* proton *polarizability* via virtual Compton scattering. First recall

	theory	expt.
$E_{0+}(\pi^0 p)(\times 10^{-3}/m_\pi)$	-1.2	$-1.31 \pm 0.08[31]$ $-1.32 \pm 0.11[32]$
$E_{0+}(\pi^0 n)(\times 10^{-3}/m_\pi)$	2.1	$1.9 \pm 0.3[33]$
$P_1/ \vec{q} (\pi p)(\times \text{GeV}^{-2})$	0.48	$0.47 \pm 0.01[31]$ $0.41 \pm 0.03[32]$

Table 4: Threshold parameters for neutral pion photoproduction.

from section 2 the concept of polarizability as the constant of proportionality between an applied electric or magnetizing field and the resultant induced electric or magnetic dipole moment—

$$\vec{p} = 4\pi\alpha_E \vec{E}, \quad \vec{\mu} = 4\pi\beta_M \vec{H} \quad (43)$$

The corresponding interaction energy is

$$E = -\frac{1}{2}4\pi\alpha_E E^2 - \frac{1}{2}4\pi\beta_M H^2 \quad (44)$$

which, upon quantization, leads to a proton Compton scattering cross section

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \left(\frac{\alpha_{em}}{m}\right)^2 \left(\frac{\omega'}{\omega}\right)^2 \left[\frac{1}{2}(1 + \cos^2 \theta)\right. \\ & \left. - \frac{m\omega\omega'}{\alpha_{em}} \left[\frac{1}{2}(\alpha_E + \beta_M)(1 + \cos \theta)^2 + \frac{1}{2}(\alpha_E - \beta_M)(1 - \cos \theta)^2 + \dots\right]\right] \end{aligned} \quad (45)$$

It is clear from Eq.(45) that, from careful measurement of the differential scattering cross section, extraction of these structure dependent polarizability terms is possible provided that

- i) the energy is large enough that these terms are significant compared to the leading Thomson piece and
- ii) that the energy is not so large that higher order corrections become important

and this has been accomplished recently at SAL and MAMI, yielding[34]

$$\alpha_E^{exp} = (12.1 \pm 0.8 \pm 0.5) \times 10^{-4} \text{fm}^3, \quad \beta_M^{exp} = (2.1 \mp 0.8 \mp 0.5) \times 10^{-4} \text{fm}^3 \quad (46)$$

A chiral one loop calculation has also been performed by Bernard, Kaiser, and Meissner and yields a result in good agreement with these measurements[35]

$$\alpha_E^{theo} = 10\beta_M^{theo} = \frac{5e^2 g_A^2}{384\pi^2 F_\pi^2 m_\pi} = 12.2 \times 10^{-4} \text{fm}^3 \quad (47)$$

The idea of *generalized* polarizability can be understood from the analogous venue of electron scattering wherein measurement of the charge form factor as a function of \bar{q}^2 leads, when Fourier transformed, to a picture of the *local* charge density within the system. In the same way the virtual Compton scattering process— $\gamma^* + p \rightarrow \gamma + p$ can provide a measurement of the \bar{q}^2 -dependent electric and magnetic polarizabilities, whose Fourier transform provides a picture of the *local polarization density* within the proton. On the theoretical side our group has performed a one loop HB χ pt calculation and has produced a closed form expression for the predicted polarizabilities[36]

$$\begin{aligned} \bar{\alpha}_E^{(3)}(\bar{q}) &= \frac{e^2 g_A^2 m_\pi}{64\pi^2 F_\pi^2} \frac{4 + 2\frac{\bar{q}^2}{m_\pi^2} - \left(8 - 2\frac{\bar{q}^2}{m_\pi^2} - \frac{\bar{q}^4}{m_\pi^4}\right) \frac{m_\pi}{\bar{q}} \arctan \frac{\bar{q}}{2m_\pi}}{\bar{q}^2 \left(4 + \frac{\bar{q}^2}{m_\pi^2}\right)}, \\ \bar{\beta}_M^{(3)}(\bar{q}) &= \frac{e^2 g_A^2 m_\pi}{128\pi^2 F_\pi^2} \frac{-\left(4 + 2\frac{\bar{q}^2}{m_\pi^2}\right) + \left(8 + 6\frac{\bar{q}^2}{m_\pi^2} + \frac{\bar{q}^4}{m_\pi^4}\right) \frac{m_\pi}{\bar{q}} \arctan \frac{\bar{q}}{2m_\pi}}{\bar{q}^2 \left(4 + \frac{\bar{q}^2}{m_\pi^2}\right)} \end{aligned} \quad (48)$$

In the electric case the structure is about what would be expected—a gradual falloff of $\alpha_E(\bar{q})$ from the real photon point with scale $r_p \sim m_\pi$. However, the magnetic generalized polarizability is predicted to *rise* before this general falloff occurs—chiral symmetry requires the presence of both a paramagnetic and a diamagnetic component to the proton. Both predictions have received some support in a soon to be announced (and tour de force) MAMI measurement at $\bar{q} = 600$ MeV[37]. However, since parallel kinematics were employed in the experiment the desired generalized polarizabilities had to be identified on top of an enormous Bethe-Heitler background. The Bates measurement, to be performed by the OOPS collaboration next spring, will take place at $\bar{q} = 240$ MeV and will use the capabilities of the OOPS detector system to provide a 90 degree out of plane measurement, which should be *much* less sensitive to the Bethe-Heitler blowtorch. We anxiously await the results.

5 Conclusion

In a short paper it is not possible to give any sense of the range of phenomena to which the concept of effective field theory as manifested via chiral perturbation theory has been applied, and interested readers can find many further applications in [16] and [28]. Nevertheless, we have tried to convey the relatively direct connection of such predictions to the underlying QCD interaction and the feature that in this way QCD itself can be tested at Bates.

Acknowledgement

It is a pleasure to acknowledge the hospitality of MIT/Bates and the organizers of this meeting. This work was supported in part by the National Science Foundation.

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